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A Net Volume Equation for Indiana

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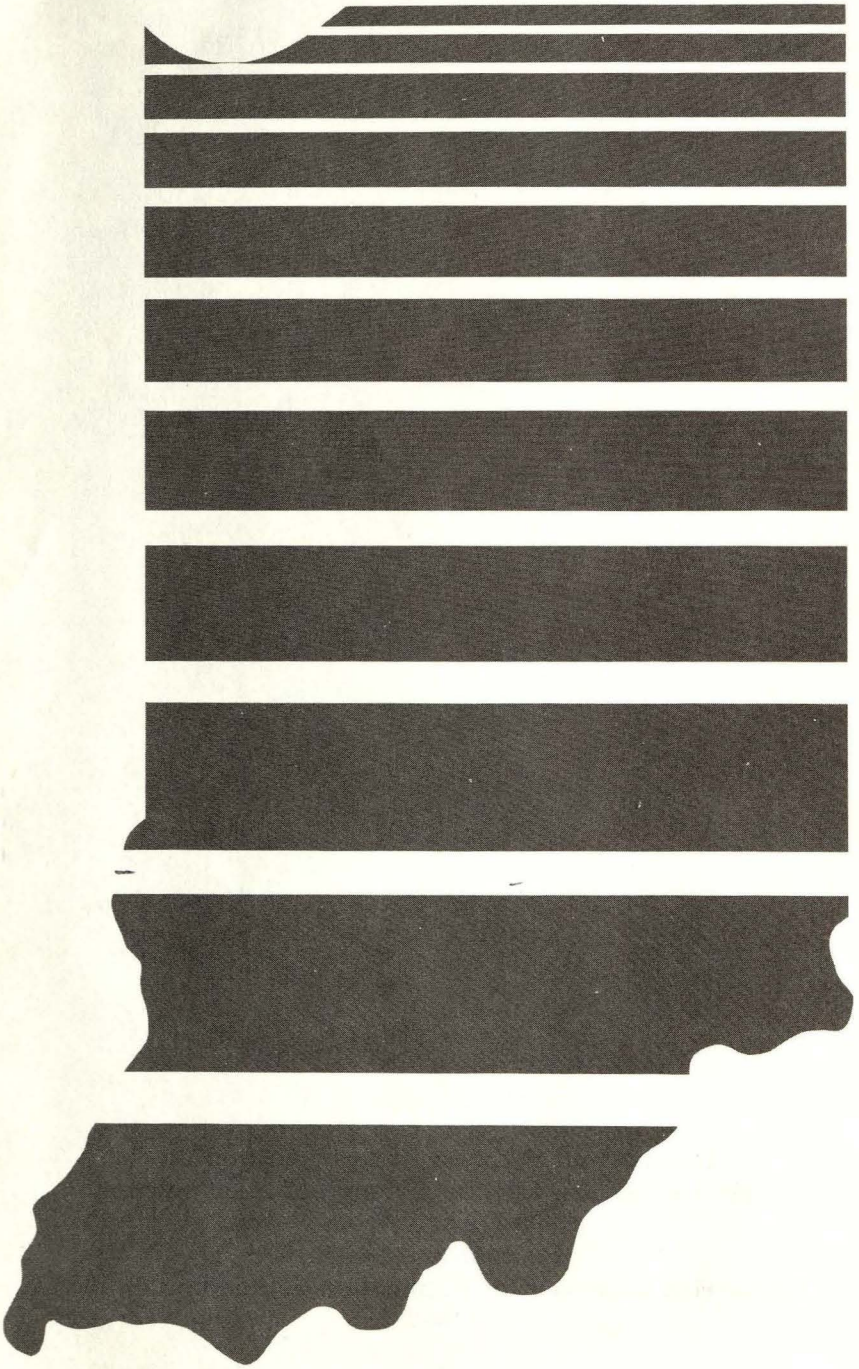

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A NET VOLUME EQUATION FOR INDIANA

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Recent development of computerized forest growth models has increased the need for accurate tree volume equations. The Stand and Tree Evaluation and Modeling System (STEMS)[1], recently calibrated for the Central States, is a computerized distance independent tree growth model. The volume equation and coefficients presented in this report will form the basis of tree volume estimates for STEMS in the eastern Central States. The equation and coefficients will also be used to estimate tree volumes in the next Indiana forest inventory to be conducted by North Central Forest Experiment Station. The equation is suitable for use on digital computers by foresters, land managers, researchers, and others in need of net volume-per-tree estimates for large forest areas. Both board foot and cubic foot volume can be estimated for most tree classes and species.

HOW THE EQUATION AND COEFFICIENTS WERE DEVELOPED

Based on Gevorkiantz and Olsen's (1955) composite volume tables for the Lake States, we used equations developed by Robert N. Stone (Appendix) to estimate the cubic foot and board foot volume in each tree. Stone's equations compute gross volume in a tree from three measurements—diameter breast height (d.b.h.), merchantable height, and top diameter outside bark (t.d.o.b.). This estimated volume was then corrected for differences in bark thickness between species and a field estimate of cull volume was subtracted to arrive at net volume.

The equation form selected for the volume regressions was a modified Weibull. It has been demonstrated that Weibull-type functions offer the wide range of flexibility necessary to accommodate a diverse set of biologically sound curve forms (Yang *et al.* 1978). The following equation form was selected for the Indiana regression:

$$V = A \cdot SI^{B} \left[1 - e^{-C \cdot DBH^{D}} \right]$$

Where the dependent variable V is net volume of the tree, and the independent variables SI and DBH are tree site index and diameter at breast height, respectively. The value of e is the constant base of natural logarithm (2.71828). A, B, C, and D are the equation coefficients. The upper asympote, $A \cdot SI^{B}$, is a function of site index. The C and D parameters are the basis of Weibull curve flexibility and are determinants of scale and shape, respectively. An example of this function can be seen graphically in fig. 1.

The equation coefficients were developed using field data collected on 12,162 trees in Indiana during the 1967 inventory. Data included d.b.h., merchantable height, t.d.o.b., site index, and tree class. For sawtimber-size trees, measurements were taken at both the sawtimber and poletimber limits of merchantability.[2] Field crews also estimated the volume of board foot and cubic foot cull in each tree (USDA Forest Service 1964).

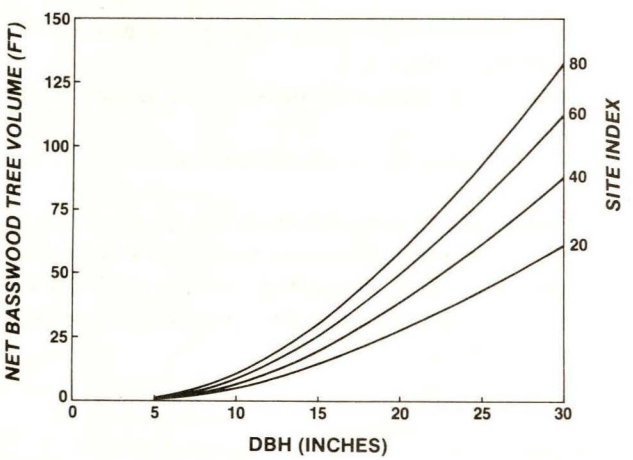

Figure 1.—*Net cubic foot volume of desirable and acceptable basswood trees by DBH and selected site index levels.*

[1]*For more information on STEMS, see: Belcher, D. L.; Holdaway M. R.; Brand G. J. A Description of STEMS: The stand and tree evaluation and modeling system. Gen. Tech. Rep. NC-79. St. Paul, MN: U.S. Department of Agriculture, Forest Service, North Central Forest Experiment Station; (in prep.)*

[2]*The poletimber merchantability limits include trees 5.0 inches d.b.h. and over, from a 1-foot stump to a minimum of 4.0-inch t.d.o.b., or to the point where the central stem breaks into limbs. Dimensions for sawtimber are 9.0-inch d.b.h. to a minimum 7-inch t.d.o.b. for softwoods and 11.0-inch d.b.h. to a minimum 9.0-inch t.d.o.b. for hardwoods.*

Cubic foot and board foot equation coefficients were developed for each of three tree class categories: "desirable and acceptable", "rough and short-log", and "rotten" (See Appendix). The groupings of species for the equations were primarily determined by the number of observations and the similarity of bole form among species. Species for which no observations were available were assigned equations on the basis of bole form. (See Appendix for species and species groups).

A computerized nonlinear regression program (Marquardt method) was used to fit the above equation to the tree data. The resulting coefficients and statistics for each regression are given in tables 1-3 (Appendix).

Because the average site index may be used when site index is unknown, the only other data needed to determine tree volumes using this model are species, d.b.h., and tree class. The average site index for Indiana species based on USDA Forest Service inventory data is found in tables 1-3 (Appendix). Note that net volume per tree varies greatly, therefore, these equations should be applied to an adequate sample over a large forest area.

HOW TO USE THE EQUATION

1. Select the appropriate species group and coefficients from tables 1-3.
2. Substitute these coefficients in the above equation.
3. Solve the equation to obtain net volume.

For example, to compute the cubic foot volume of a 15-inch acceptable basswood tree with a site index of 60 we would first obtain the coefficients for basswood from table 1. Substituting into the general net volume equation:

$$V = A \cdot SI^{B}\left[1.0\text{-}e^{-C \cdot DBH^{D}}\right]$$

$$V = 16.882\ (60.0)^{.572}\left[1.0\text{-}e^{-.0001(15.0)^{2.707}}\right]$$

$$V = (16.882) \cdot (10.402) \cdot (1.0\text{-}.859)$$

$V = 24.85$ cubic feet

This volume may be verified graphically in figure 1.

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APPENDIX

VOLUME EQUATIONS

Stone uses the following dependent variables in his equation to estimate cubic foot volume (cv): diameter at breast height (D), merchantable height (H), and diameter outside bark at merchantable height (T).

$$CV = (3.0086 \times 10^{-3} + S1 + S2 + S3 + S4 + S5 + S6 + S7 + S8) \cdot 79.0$$

where

$S1 = (2.0355 \times 10^{-3})D$,
$S2 = (-3.0018 \times 10^{-3})T$,
$S3 = (6.2381 \times 10^{-5})D^2$,
$S4 = (2.5705 \times 10^{-5})D^2 \cdot H$,
$S5 = (-7.0090 \times 10^{-6})H^2$,
$S6 = (3.6708 \times 10^{-5})H \cdot T^2$,
$S7 = (8.1400 \times 10^{-10})D^2 \cdot H^3$, and
$S8 = (-1.90000 \times 10^{-9})D^2 \cdot H^2 \cdot T$.

Stone's board foot volume equation as modified by Hahn (1975, 1976) is as follows:

$$BV = 17.7488 + 7.3846 \cdot CV - 2.3523 \cdot D - .89945 \cdot H + 2.0726 \cdot T$$

where

BV = board foot volume (International 1/4-inch rule).

DEFINITION 0F TERMS

Desirable trees.—Live trees that have no serious defects limiting present or prospective use, have relatively high vigor, and have no visible signs of pathogens that may result in death or serious deterioration before rotation age. They would be favored by forest managers in silvicultural operations.

Acceptable trees.—Live trees that have no serious defects limit present or prospective use but that have pathogens or damage affecting quality.

Rough trees.—Live trees that do not contain at least one merchantable 12-foot log, now or prospectively, because of roughness, poor form, or noncommercial species.

Rotten trees.—"Rough" trees in which more than 50 percent of the cull volume is rotten.

Short-log trees.—Live sawtimber sized trees that contain at least one merchantable from 8- to 11-foot saw log but not a 12-foot saw log, now or prospectively.

PRINCIPAL TREE SPECIES OF INDIANA[3]

SOFTWOOD SPECIES

Pines
- Shortleaf pine ..... *Pinus echinata*
- Virginia pine ..... *Pinus virginiana*
- Red pine ..... *Pinus resinosa*
- White pine ..... *Pinus strobus*
- Scotch pine ..... *Pinus sylvestris*

Other softwoods
- Eastern redcedar ..... *Juniperus virginiana*
- Tamarack ..... *Larix laricina*
- Baldcypress ..... *Taxodium distichum*

HARDWOOD SPECIES

White oaks
- White oak ..... *Quercus alba*
- Swamp white oak ..... *Quercus bicolor*
- Bur oak ..... *Quercus macrocarpa*
- Swamp chestnut oak ..... *Quercus michauxii*
- Chinkapin oak ..... *Quercus muehlenbergii*
- Chestnut oak ..... *Quercus pinus*
- Post oak ..... *Quercus stellata*

Red oaks
- Northern red oak ..... *Quercus rubra*
- Cherrybark oak ..... *Quercus falcata* var. *pagodaefolia*
- Shumard oak ..... *Quercus shumardii*
- Black oak ..... *Quercus velutina*
- Scarlet oak ..... *Quercus coccinea*
- Southern red oak ..... *Quercus falcata*
- Shingle oak ..... *Quercus imbricaria*
- Pin oak ..... *Quercus palustris*

Hickories
- Mockernut hickory ..... *Carya tomentosa*
- Shagbark hickory ..... *Carya ovata*
- Shellbark hickory ..... *Carya laciniosa*
- Pecan ..... *Carya illinoensis*
- Pignut hickory ..... *Carya glabra*
- Bitternut hickory ..... *Carya cordiformis*

Hard maple ..... *Acer saccharum*

Soft maples
- Red maple ..... *Acer rubrum*
- Silver maple ..... *Acer saccharinum*
- Boxelder ..... *Acer negundo*

Beech ..... *Fagus grandifolia*

Sweetgum ..... *Liquidambar styraciflua*

Blackgum ..... *Nyssa sylvatica* var. *biflora*

Ashes
- White ash ..... *Fraxinus americana*
- Black ash ..... *Fraxinus nigra*
- Green ash ..... *Fraxinus pennsylvanica*
- Blue ash ..... *Fraxinus quadrangulata*

Cottonwood ..... *Populus deltoides*

Aspens
- Balsam poplar ..... *Populus balsamifera*
- Bigtooth aspen ..... *Populus grandidentata*
- Quaking aspen ..... *Populus tremuloides*

Basswood ..... *Tilia americana*

Yellow-poplar ..... *Liriodendron tulipifera*

Walnuts
- Black walnut ..... *Juglans nigra*
- Butternut ..... *Juglans cinerea*

Black cherry ..... *Prunus serotina*

[3]*The common and scientific names are based on: Little, Elbert L. Check list of native and naturalized trees of the United States. Agric. Handbk. 541. Washington, D.C.: U.S. Department of Agriculture, Forest Service; 1979. 375 p.*

Elms
- Winged elm ..... *Ulmus alata*
- American elm ..... *Ulmus americana*
- Siberian elm ..... *Ulmus pumila*
- Slippery elm ..... *Ulmus rubra*
- Rock elm ..... *Ulmus thomasii*

Sycamore ..... *Platanus occidentalis*

Other hardwoods
- Yellow birch ..... *Betula alleghaniensis*
- River birch ..... *Betula nigra*
- Paper birch ..... *Betula papyrifera*
- Ohio buckeye ..... *Aesculus glabra*
- Hackberry ..... *Celtis occidentalis*
- Northern catalpa ..... *Catalpa speciosa*
- Flowering dogwood ..... *Cornus florida*
- Common persimmon ... *Diospyros virginiana*
- Honeylocust ..... *Gleditsia triacanthos*
- Kentucky coffeetree ... *Gymnocladus dioicus*
- Osage-orange ..... *Maclura pomifera*
- Cucumbertree ..... *Magnolia acuminata*
- Black locust ..... *Robinia pseudoacacia*
- Black willow ..... *Salix nigra*
- Sassafras ..... *Sassafras albidum*

METRIC EQUIVALENTS

1,000 board feet (International 1/4-inch log rule) = 3.48 cubic meters.

Breast Height = 1.37 meters above the ground .1 cubic foot = 0.0283 cubic meter.

1 foot = 30.48 centimeters or 0.3048 meter.

1 inch = 25.4 millimeters or 2.54 centimeters or 0.0254 meter.

Table 1.--Net cubic foot volume regression coefficients by tree class and species group[1/], Indiana, 1982

Species group	Coefficient				Average site index	D.b.h.		Observations	Standard error
	A	B	C	D		Range	Average	Number	Cubic feet
DESIRABLE AND ACCEPTABLE TREES									
Pines	32.7134	0.3785	0.000089	2.7986	72	5-19	8	94	0.4
Other softwoods	24.3201	.8216	.000044	2.5616	65	5-44	14	101	.7
White oaks	21.1498	.5754	.000110	2.5942	66	5-45	14	2,281	.4
Red oaks	126.1555	.2152	.000081	2.5973	71	5-44	15	2,305	.5
Hickories	24.3795	.5106	.000086	2.7597	69	5-34	11	1,576	.4
Basswood	16.8823	.5723	.000100	2.7071	72	5-30	14	119	.4
Beech	252.1400	.0336	.000110	2.5349	79	5-33	16	427	.6
Hard and soft maples	948.9211	0	.000059	2.3163	70	5-45	12	1,436	.4
Elms	341.7616	0	.000083	2.5673	75	5-27	11	375	.3
Ashes	168.5280	.0904	.000099	2.6121	76	5-32	11	761	.3
Sycamore	30.7110	.5829	.000073	2.6184	80	5-38	16	398	.6
Cottonwood-willow	14.1604	.8812	.000056	2.4651	80	5-43	16	254	.6
Aspen-birch	32.0897	.4262	.000177	2.4820	84	5-29	11	123	.3
Sweetgum	12.3238	.5555	.000115	2.8126	83	5-29	12	180	.3
Blackgum	15.8460	.5938	.000063	2.8574	76	5-28	13	125	.4
Black cherry	21.9481	.4170	.000263	2.5539	75	5-25	11	195	.3
Walnuts	65.4786	.0383	.000265	2.7537	78	5-22	11	314	.3
Yellow poplar	23.6994	.5052	.000097	2.7495	79	5-35	14	582	.5
Other hardwoods	16.7211	.5991	.000080	2.7109	77	5-37	10	516	.3
ROUGH AND SHORT-LOG TREES									
Softwoods	22.7769	.4240	.000057	2.9092	70	5-20	9	37	.2
White oaks	104.1617	.0890	.000119	2.5916	65	5-41	18	221	.6
Red oaks	21.1882	.6788	.000073	2.4026	70	5-46	16	204	.5
Hickories	18.6983	.4097	.000132	2.7085	70	5-27	13	113	.3
Beech	4.0681	.5642	.000099	3.2936	82	5-30	13	78	.4
Hard and soft maples	72.0477	.0743	.000166	2.7068	70	5-40	14	244	.4
Elms	25.5502	.1927	.000200	2.7686	73	5-27	10	122	.2
Ashes	5.0944	.6960	.000158	2.6118	75	5-32	12	111	.3
Sycamore	33.2901	.4860	.000050	2.7414	77	5-34	15	44	.4
Cottonwood-willow	62.0266	.4397	.000037	2.6637	72	5-30	14	49	.4
Black cherry	8.9688	.3930	.000253	2.8160	72	5-27	10	72	.3
Other hardwoods	57.1662	.4545	.000048	2.5027	76	5-47	10	234	.3
Noncommercial species	42.6350	0	.000454	2.5315	75	5-23	8	68	.2
ROTTEN TREES									
All species	13.5690	.7493	.000054	2.4058	73	5-62	19	524	1.0

[1/] See Appendix III for individual species within major groups.

Table 2.--Net board foot[1/] saw log volume regression coefficients by tree class and species group[2/], Indiana, 1982

Species group	Coefficient				Average site index	D.b.h.		Observations	Standard error
	A	B	C	D		Range	Average	Number	Board feet
DESIRABLE AND ACCEPTABLE TREES									
Pines	68.6528	0.9374	0.000040	2.6643	69	10-38	17	94	3.2
Other softwoods	68.6358	.6359	.000082	2.8486	71	11-25	16	48	2.8
White oaks	103.7664	.6707	.000076	2.6670	73	11-44	16	1,485	3.6
Red oaks	871.4258	.0682	.000112	2.5879	68	11-45	18	1,686	3.0
Hickories	1,151.2737	0	.000077	2.7331	71	11-44	18	745	3.2
Basswood	68.6283	.6229	.000045	3.0375	71	11-26	16	75	3.0
Beech	343.5727	.3245	.000081	2.6745	69	11-33	15	326	2.7
Hard and soft maples	68.6272	.5665	.000140	2.7006	74	11-34	17	747	3.5
Elms	693.5429	.0476	.000071	2.9203	68	11-40	16	138	2.4
Ashes	68.6341	.7813	.000079	2.5203	73	11-45	18	348	3.6
Sycamore	68.6398	.4570	.000112	2.9491	77	11-27	15	272	2.6
Cottonwood	68.6298	.6536	.000053	2.8696	79	11-33	17	182	4.2
Aspen-birch	68.6338	.6326	.000067	2.8446	80	11-35	21	49	5.2
Sweetgum	68.6313	.8876	.000052	2.4993	82	11-43	21	89	3.7
Blackgum	178.1367	.3166	.000083	2.8811	79	11-31	17	75	2.3
Black cherry	68.6298	.5869	.000045	3.0054	81	11-37	16	84	2.9
Walnut	68.6472	.5722	.000087	2.8488	75	11-28	15	155	2.9
Yellow poplar	261.7747	.3820	.000088	2.6472	79	11-35	16	392	3.3
Other hardwoods	164.5671	.4052	.000061	2.8766	77	11-35	16	160	2.7
SHORT-LOG TREES									
All species	1,454.7622	0	.000133	2.2236	68	11-41	19	524	.8

[1/] International 1/4-inch rule.

[2/] See Appendix III for individual species within major groups.

Table 3.--Net cubic foot saw log volume regression coefficients by tree class and species group[1/], Indiana, 1982

Species group name	Coefficient				Average site index	D.b.h.		Observations	Standard error
	A	B	C	D		Range	Average		
								Number	Cubic feet
DESIRABLE AND ACCEPTABLE TREES									
Pines	8.5339	0.5167	0.000231	2.7641	69	10-38	17	270	0.2
Other softwoods	32.7134	.3785	.000089	2.7986	71	11-25	16	48	.4
White oaks	32.6541	.4767	.000084	2.6689	73	11-44	16	1,485	.5
Red oaks	108.1743	.0818	.000175	2.4623	68	11-45	18	1,686	.5
Hickories	142.5035	.0034	.000116	2.6411	71	11-44	18	745	.5
Basswood	34.0091	.2952	.000025	3.2982	71	11-26	16	75	.5
Beech	1,173.2462	0	.000025	2.4259	69	11-33	15	326	.4
Hard and soft maples	190.0165	0	.000199	2.3569	74	11-34	17	747	.5
Elms	77.4351	.0922	.000154	2.6564	63	11-40	16	138	.4
Ashes	527.1757	0	.000078	2.2867	73	11-45	18	348	.5
Sycamore	43.4212	.1437	.000141	2.8151	77	11-27	15	272	.4
Cottonwood	75.3457	.4188	.000016	2.8728	79	11-30	17	182	.6
Aspen-birch	51.9064	.4039	.000064	2.5744	80	11-35	21	49	.5
Sweetgum	65.2252	.4613	.000060	2.4471	82	11-43	21	89	.5
Blackgum	4.0386	.6312	.000064	3.1986	79	11-31	17	75	.4
Black cherry	50.0470	.2127	.000064	2.8882	81	11-37	16	84	.5
Walnuts	528.3431	.1433	.000030	2.4206	75	11-28	15	155	.4
Yellow poplar	26.6412	.4204	.000135	2.5659	79	11-35	16	392	.5
Other hardwoods	15.0211	.4755	.000087	2.8131	77	11-35	16	160	.4
SHORT-LOG TREES									
All species	2.3840	.6855	.000083	2.7726	68	11-41	19	524	.4

[1/] See Appendix III for individual species within major groups.

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A net volume equation for Indiana. Res. Bull. NC-63. St. Paul, MN: U.S. Department Agriculture, Forest Service, North Central Forest Experiment Station; 1982. 7 p.

Describes a Weibull-type volume equation for Indiana developed as part of the ongoing Resource Evaluation research in the Central States. Equation coefficients are presented by species groupings for both cubic foot and board foot volumes for three tree class categories.

KEY WORDS: Inventory, computer, growing-stock, cull, cubic foot, board foot, Weibull.

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